Mechanical and
Aerospace
Engineering

NSTX-U

PPPL

Neural net modeling of equilibria in NSTX-U

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Background and definitions:

- **plasma equilibrium:**

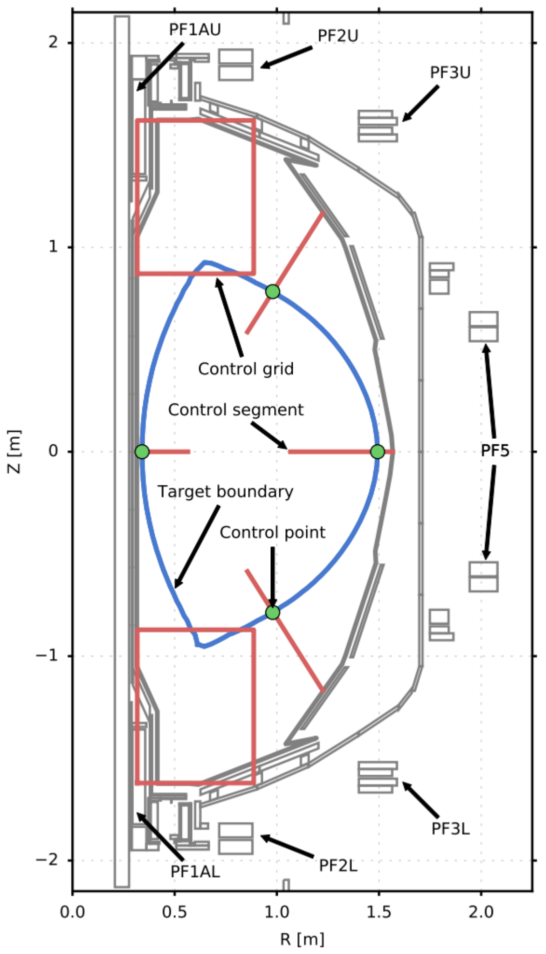
Description of the {plasma position, magnetic flux surfaces, currents, internal pressure, etc.} needed to characterize force balance.

- **shape control:**

Model for controlling the plasma's position and boundary shape, using external coils as actuators.

- **neural network (NN):**

Computational tool that uses data to learn complex functions. Can approximate complex calculations *quickly* and *accurately*.



NSTX-U shape control elements. [1]

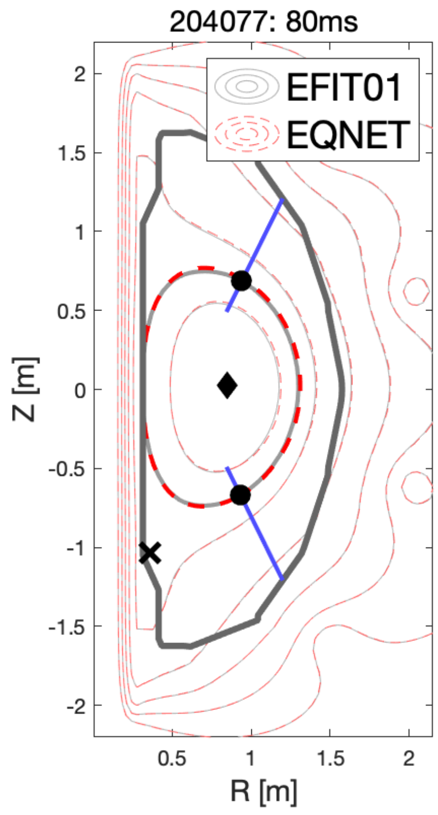
Trained two neural networks relevant to plasma control

Eqnet: NN capable of predicting the plasma equilibrium from either: diagnostics, or coil currents + plasma internal profiles. Trained on EFIT01 [13,14] data.

Pertnet: NN capable of predicting the *plasma response*—a critical component of the shape control model. Trained on Gspert code [15,16].

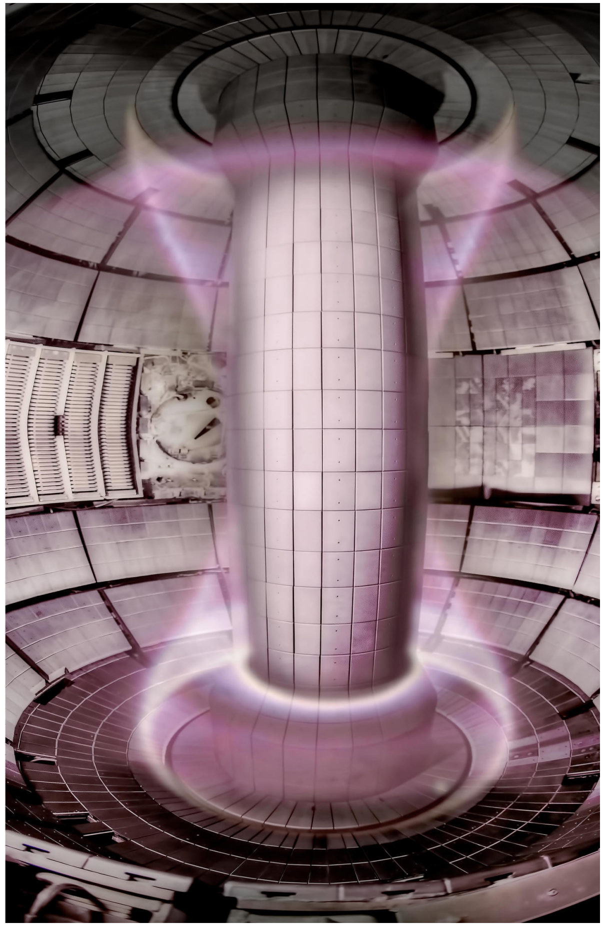
Motivation: Physics-based models for equilibrium design/estimation/control not fast enough or applicable to all desired use cases.

J.T. Wai, M.D. Boyer, E. Kolemen, "Neural net modeling of equilibria in NSTX-U", *submitted to Nuclear Fusion*, 2022.



NNs will be integrated into several desired control applications

- **Between shot scenario design (Eqnet + Pertnet):**
  - Fast plasma simulator can inform physics operators, enables numerical optimization of actuator trajectories.
  - Current ‘flight simulators’ not fast enough to run between shots.
  - Integrate & supplement previous NN profile predictors [1,2]
- **Increased availability of equilibrium reconstruction (Eqnet):**
  - Real-time EFIT reconstruction unavailable early in the shot. Workaround uses a hybrid gap controller and isoflux controller but suffers oscillations during controller transfer.
  - Simpler method to use a single isoflux controller – measurements can be supplied by NN when rt-EFIT not available.
- **Continuously monitor and update shape controller (Pertnet):**
  - Monitor vertical growth rate directly (as opposed to proxy parameters like elongation) and take corrective action if too unstable
  - Provide real-time updates of the shape control model for better controller performance



NSTX-U tokamak with plasma. Source: pppl.gov

Eqnet: NN plasma equilibrium solver

Plasma equilibrium describes force balance condition

- The equilibrium describes the magnetic field structure of the plasma. Balance between outward pressure force and inward JxB force.

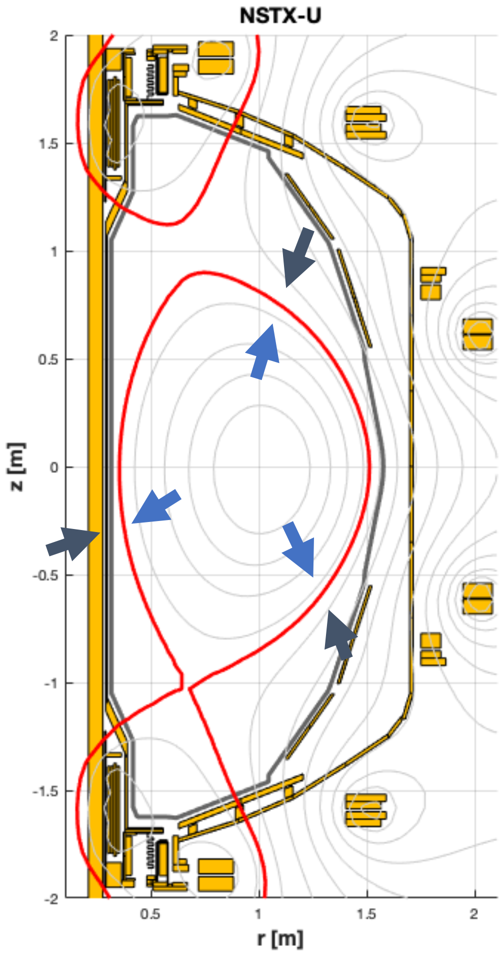
- In a tokamak, the equilibrium is described by the Grad-Shafranov equation

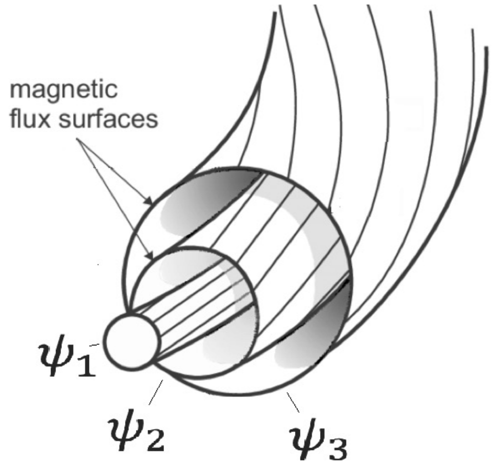
$$J \times B = \nabla P$$

$$-\mu_0 r J_\phi = r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right) + \frac{\partial^2 \psi}{\partial z^2}$$

$$J_\phi = J_\phi^{pla} + J_\phi^{ext}$$

$$J_\phi^{pla} = RP'(\psi) + \frac{FF'(\psi)}{\mu_0 R}$$

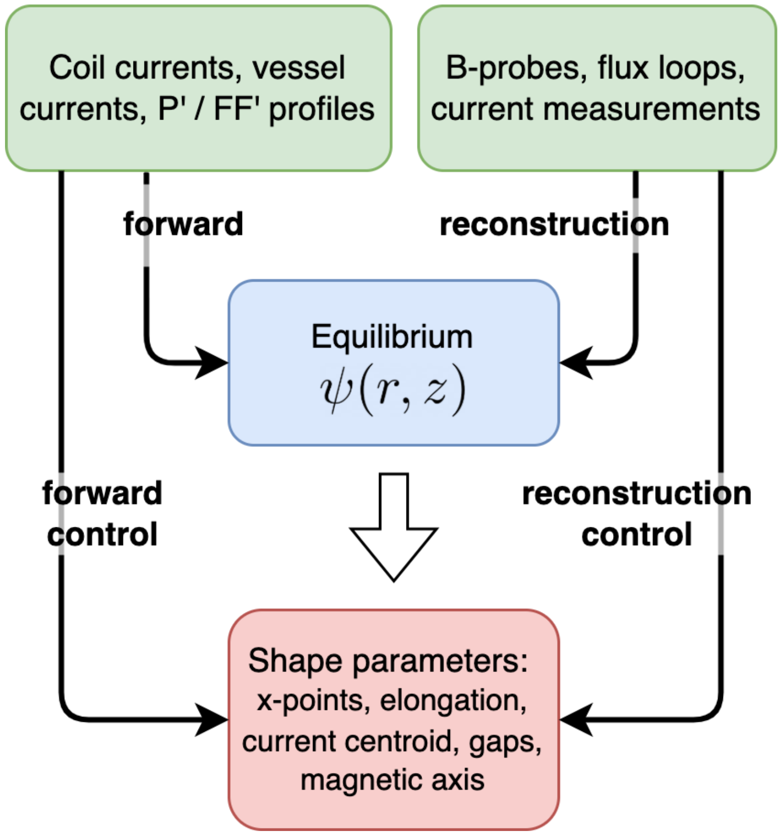




Magnetic field lines lying on flux surfaces.

Eqnet: a neural network equilibrium solver

- Solving the free-boundary Grad-Shafranov equation is a nonlinear, iterative computation.
  - Iterate between updating the plasma current, externally applied field, and flux.
- Eqnet is a neural net version of this process. Can work in different modes (data inputs and outputs): **forward solver** and **reconstruction**.
- Also trained to predict parameters directly, for use as a control estimator. Similar to previous NNs [3-11]

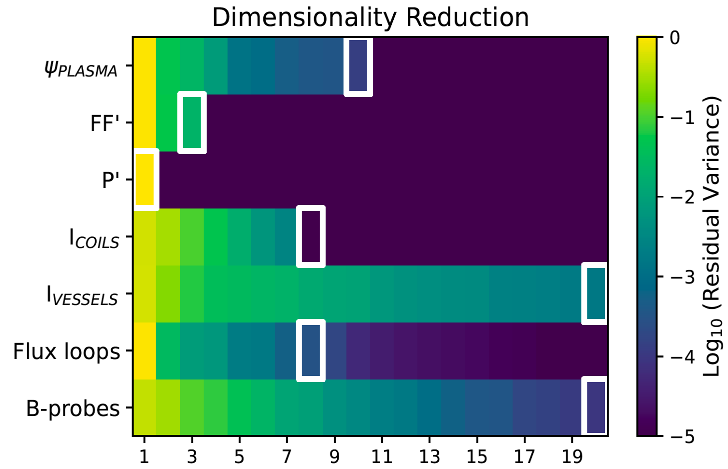


Data inputs and outputs are compressed using Principal Component Analysis (PCA)

- Data inputs and outputs are compressed using PCA to capture 99.5% variance
- Goal is to predict equilibrium ψ but more accurate to predict ψ^{pla} since applied flux is known.

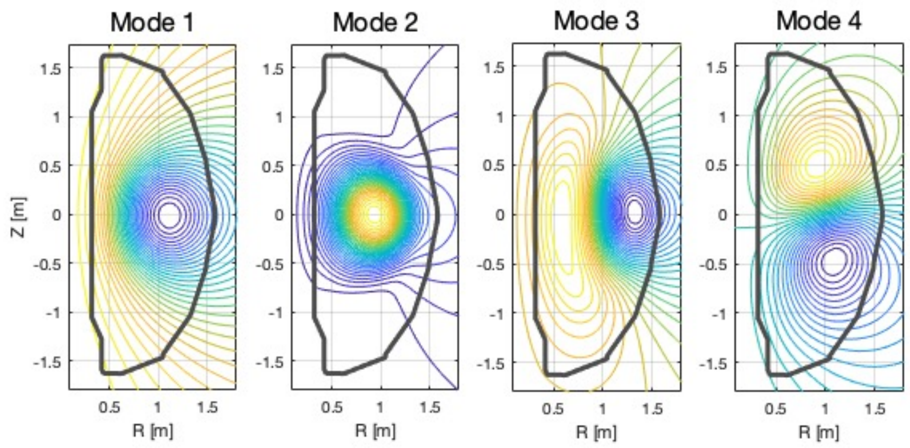
$$\psi = \psi^{pla} + \psi^{app}$$
$$\psi^{app} = M_{pc} I_c + M_{pv} I_v$$

- PCA on ψ^{pla} : (65x65 grid) → 11 modes
  - Results are intuitive and can match a wide range of equilibria (limited, lower null, upper null, off-normal)

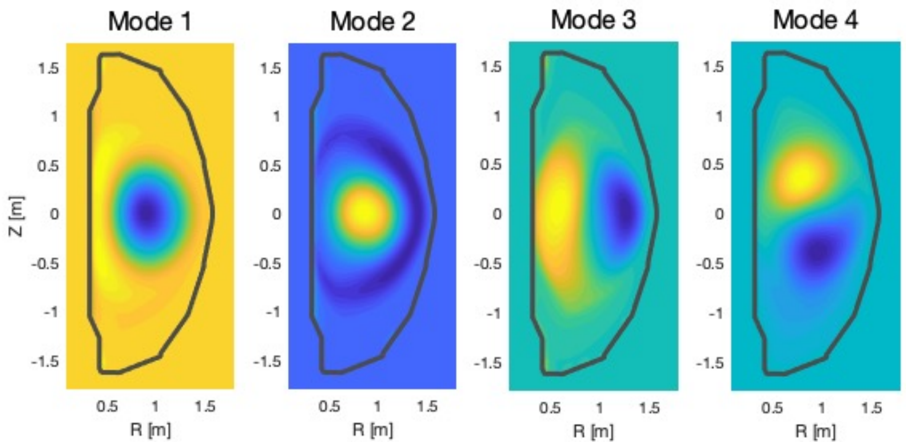


Number of PCA components used. Outlined box indicates >99.5% captured variance.

Plasma Flux PCA Modes



Plasma Current PCA Modes



Eqnet architecture is based on a fully-connected NN

- Neural network – tool for approximating many types of functions $y = f(x)$. Has many parameters or weights, w. During training weights are tuned so that predicted outputs match target outputs.

$$\min_{w} J(w) = [f(x_{train}, w) - y_{train}]^2$$

X_{train} = coil currents, vessel currents, P' / FF' profiles

$Y_{train} = \psi^{pla}$

- Data is from EFIT01 [13,14] magnetics-only reconstruction of 2015-2016 NSTXU campaign. 220 shots & 25,000 equilibria
- Divided 80-10-10 by shot number into train, validation, test datasets

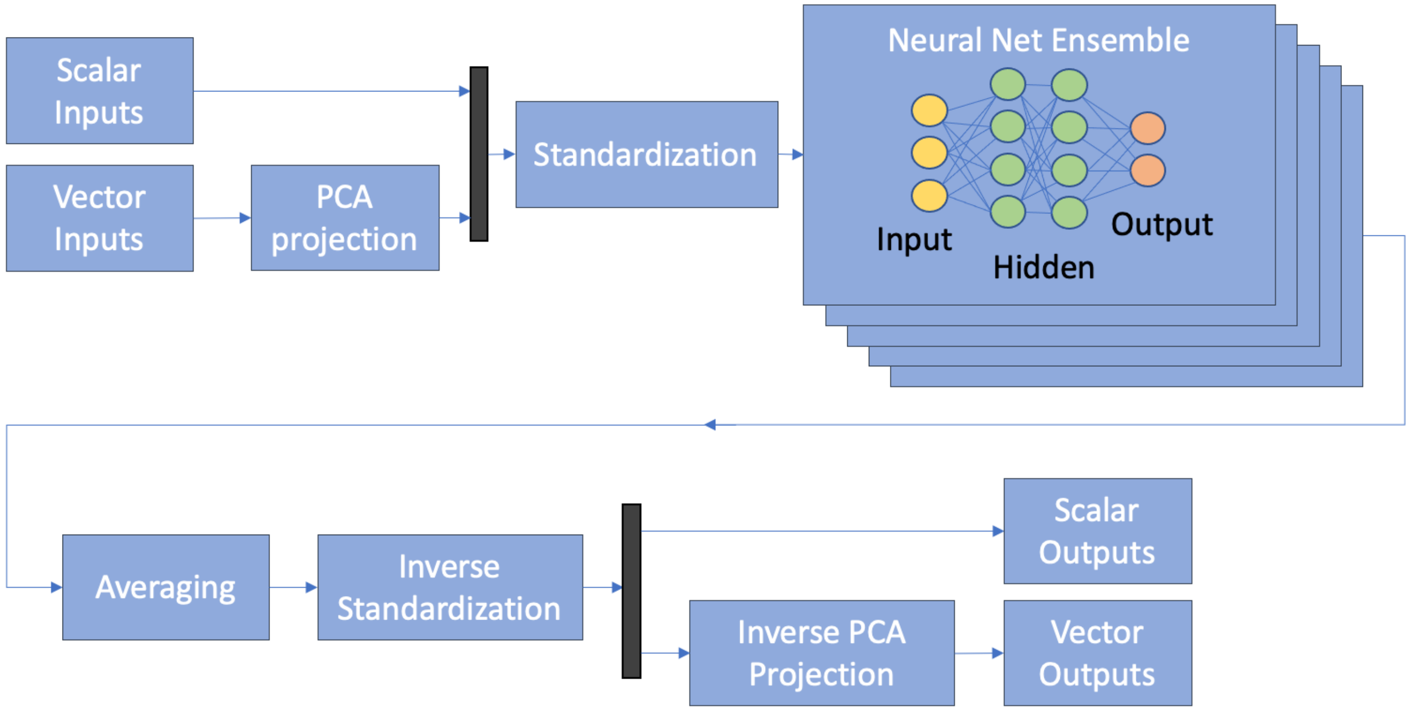
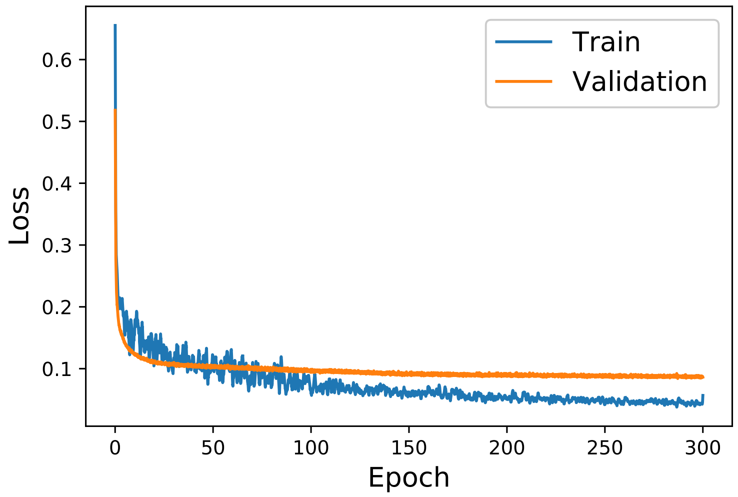


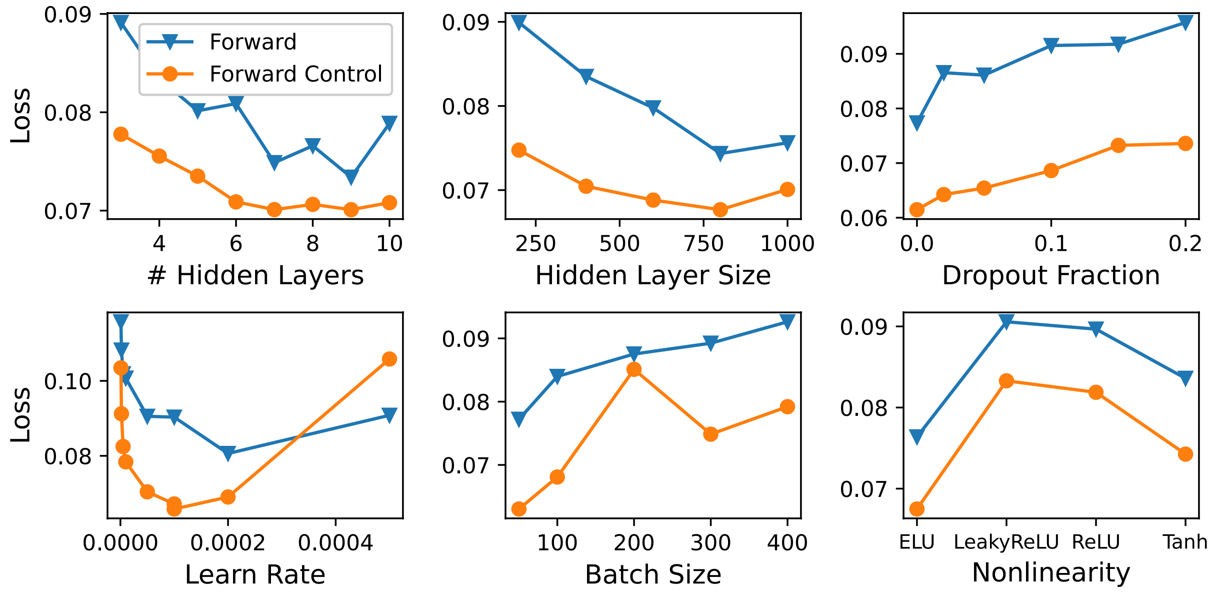
Fig 1: Neural net and data architecture. The core of the model is a fully-connected NN.

Grid scans used to optimize hyperparameters

- Hyperparameters of the neural network found by performing grid scans
- Final result is a fully-connected network
  - 8 hidden layers, size 800, ELU activation

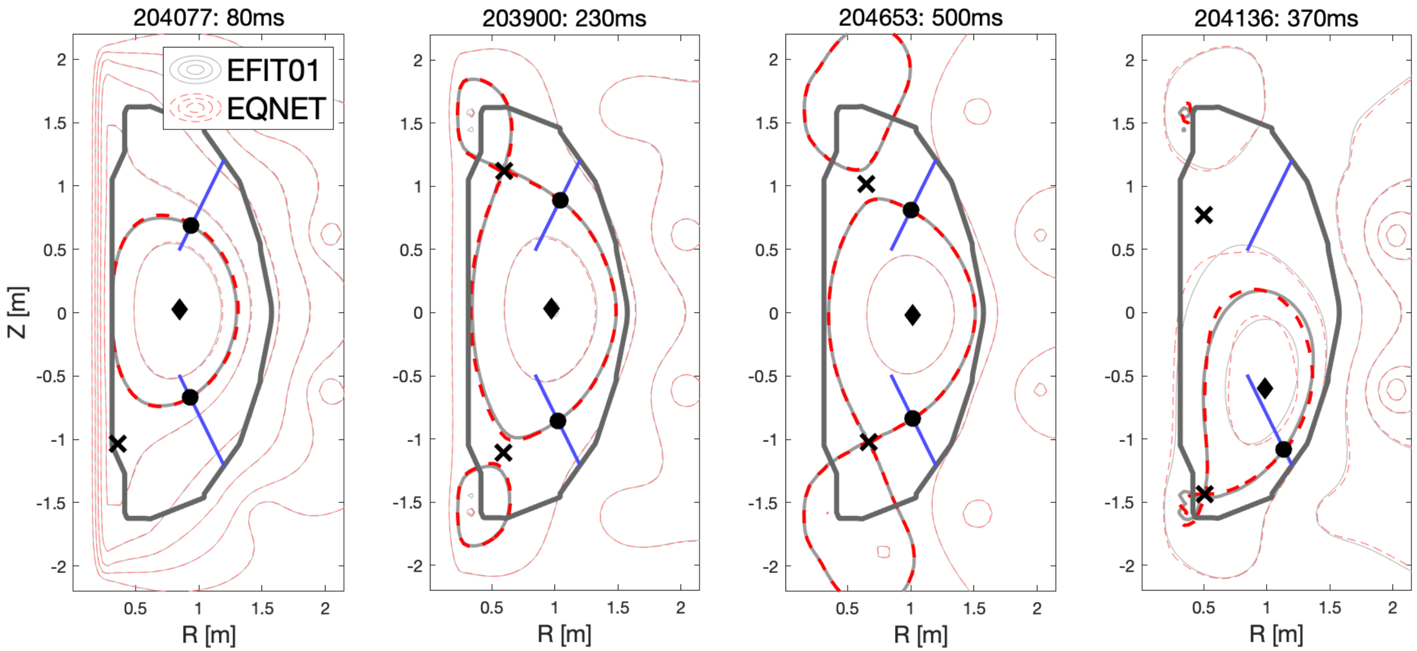


Typical loss curve from NN training

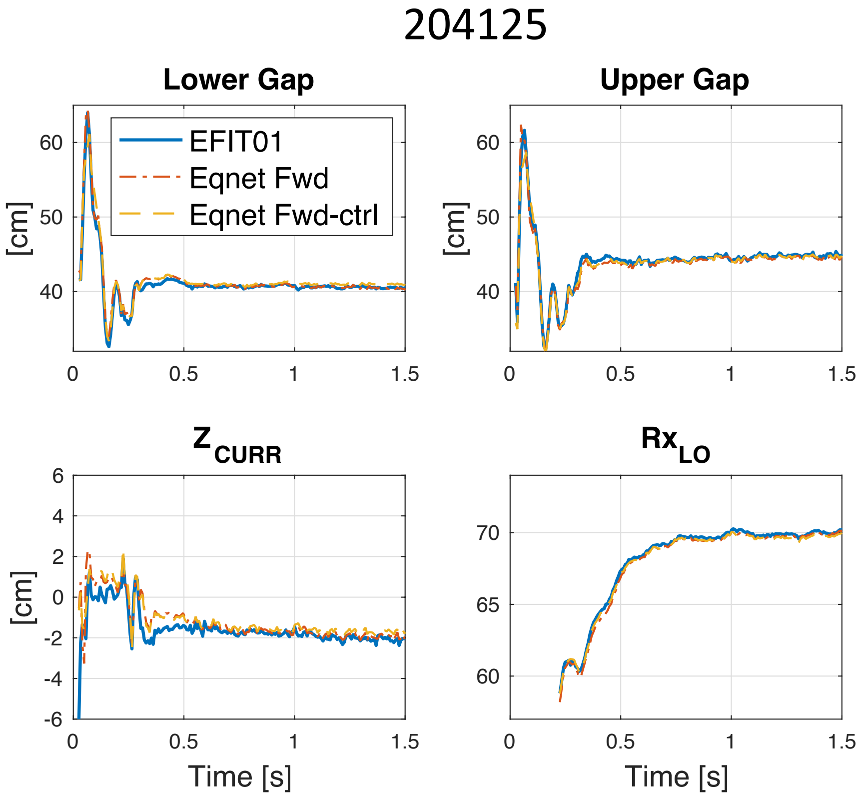


Hyperparameter grid scans

High agreement of flux surfaces is verified visually



Flux surfaces predictions capture a wide range of equilibria with high accuracy (median x-point error 4mm)



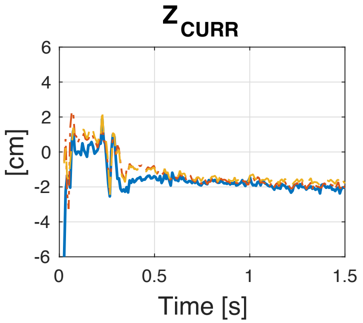
Shape parameter predictions.
Eqnet Fwd = parameters extracted from NN flux predictions.
Eqnet Fwd-Ctrl = parameters predicted directly by NN.

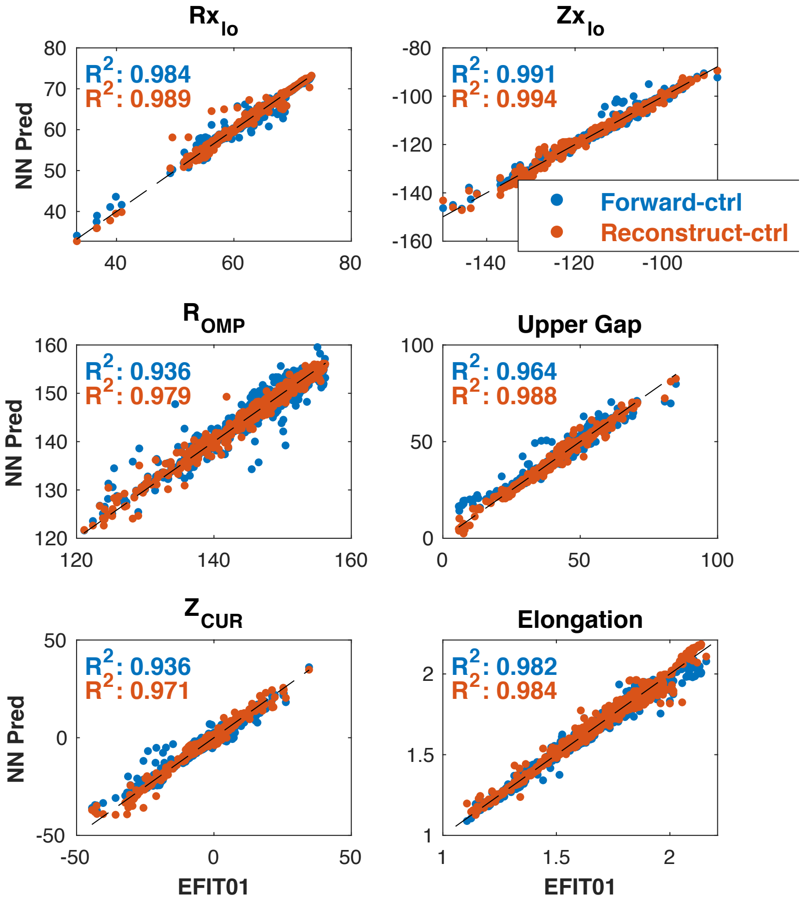
Eqnet predictions of features have errors <1cm

Majority of predictions <1cm and within the range of EFIT01 accuracy

Multiple-cm errors do occur:

- larger than desired for control and simulation purposes
- however, these happen during oscillatory dynamic phases (e.g. 68% errors >3cm occur within first or last 100ms)





Shape parameter prediction performance				
	Rx, lo	Zx, lo	Rcur	Zcur
Root-Mean-Squared-Error	5.6mm	7.5mm	9.0mm	9.0mm

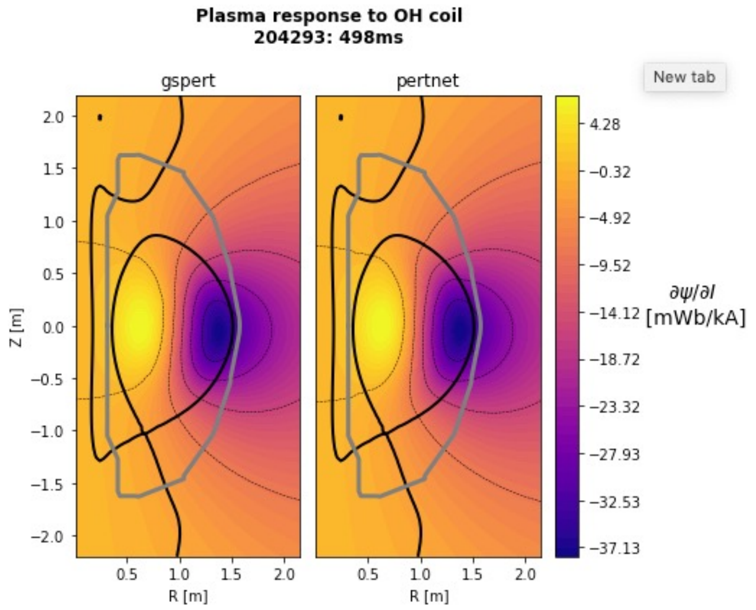
Pertnet: NN calculation of the plasma response

The plasma response describes how the plasma redistributes in response to external perturbations

- If a coil current is perturbed slightly, it changes the magnetic field so that there is a force exerted on the plasma. Response = how does the plasma current shift and redistribute in response to this force.

$$\frac{\partial J_\phi^{pla}}{\partial x} \Leftrightarrow \frac{\partial \psi^{pla}}{\partial x}$$

$$x \in (\text{coil currents, vessel currents, } l_i,\ \beta_p)$$



- Plasma response is an integral part of the shape control circuit model, which describes the evolution of coil and vessel currents.

$$v = RI + \dot{\psi}$$

$$= RI + M\dot{I} + M_{cp}\frac{\partial J_\phi^{pla}}{\partial I}\dot{I}$$

- Also needed to describe how shape parameters respond to actuators.

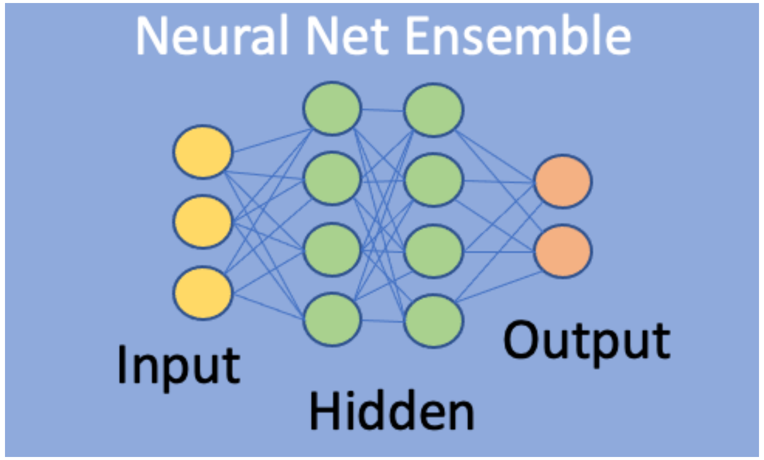
$$y = f(\psi(x))$$

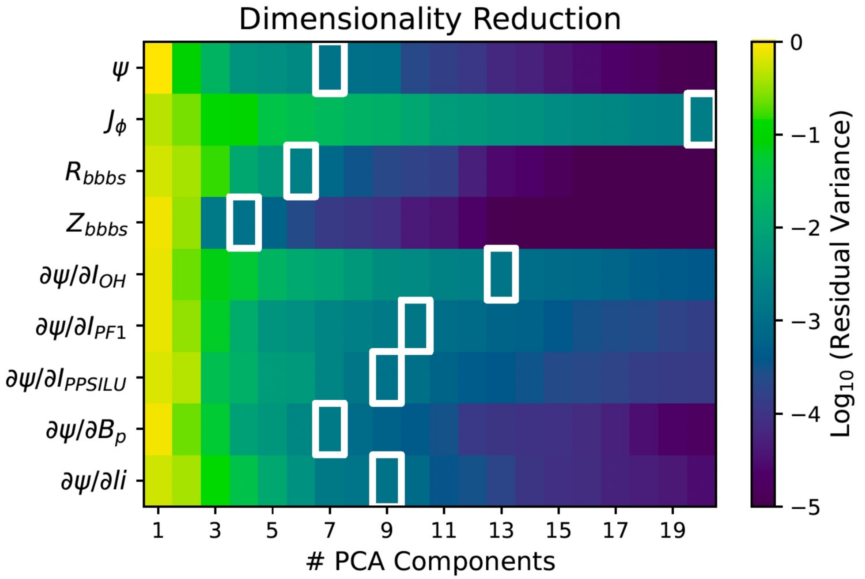
$$\frac{\partial y}{\partial x} = \frac{\partial f}{\partial \psi}\left(\frac{\partial \psi^{ext}}{\partial x} + \frac{\partial \psi^{pla}}{\partial x}\right)$$

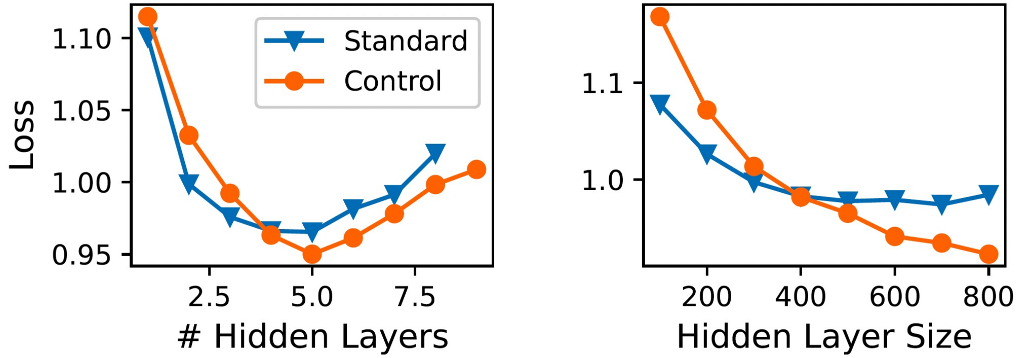
- Can also be used to calculate the growth rate of the plasma vertical instability (related to eigenvalue of circuit equation).

Pertnet utilizes similar data and network architecture to Eqnet

- Ground truth is taken from **gspert** code [15,16] applied to EFIT01 equilibria.
- Similar data and NN architecture to Eqnet. Input is a full description of the equilibrium.

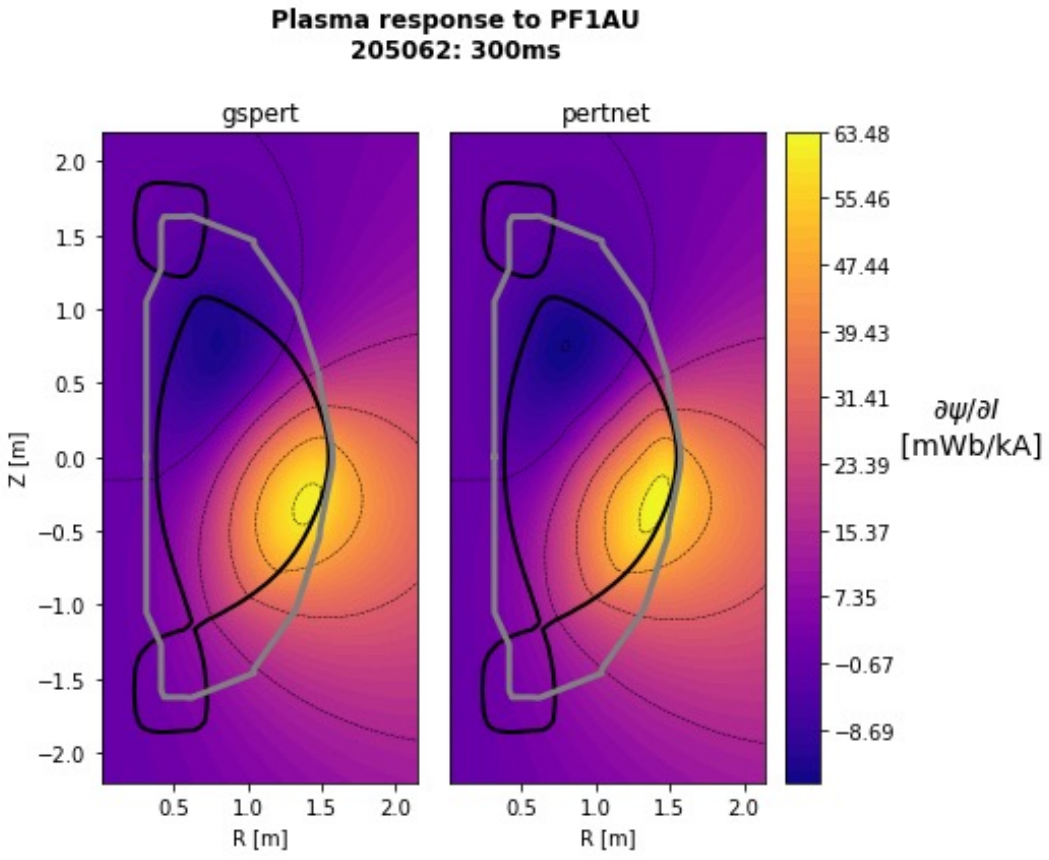


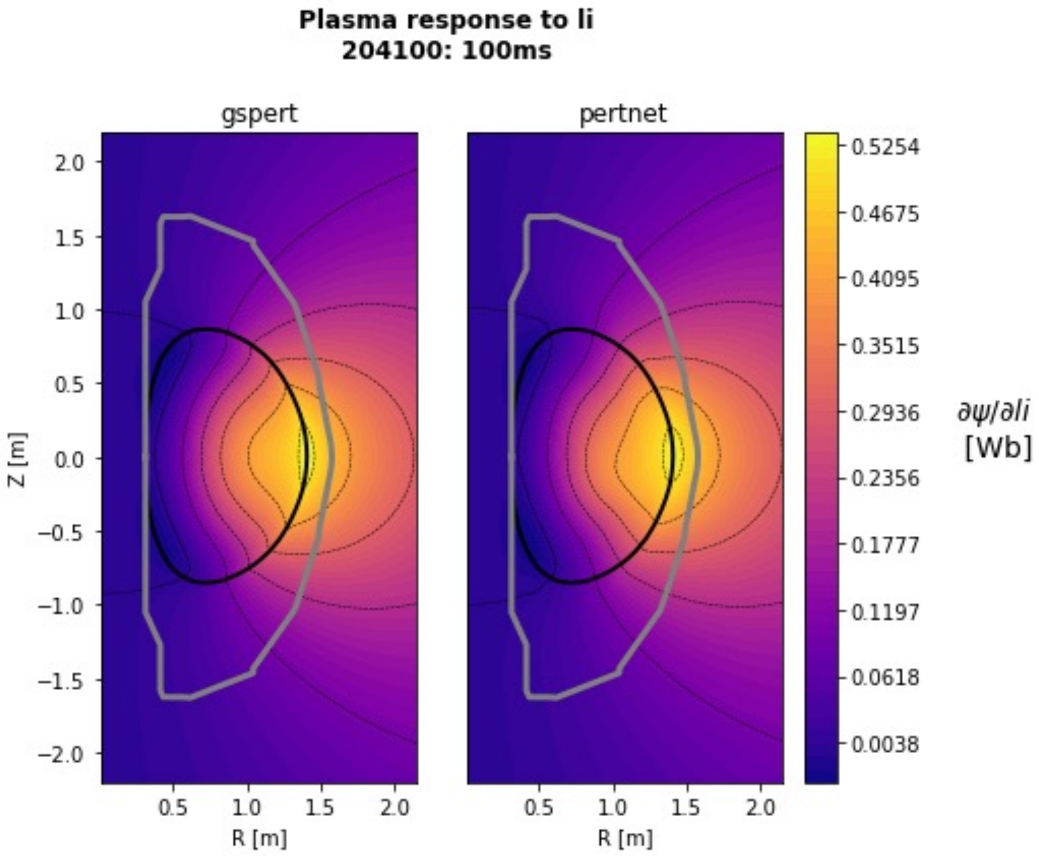




Pertnet hyperparameter tuning

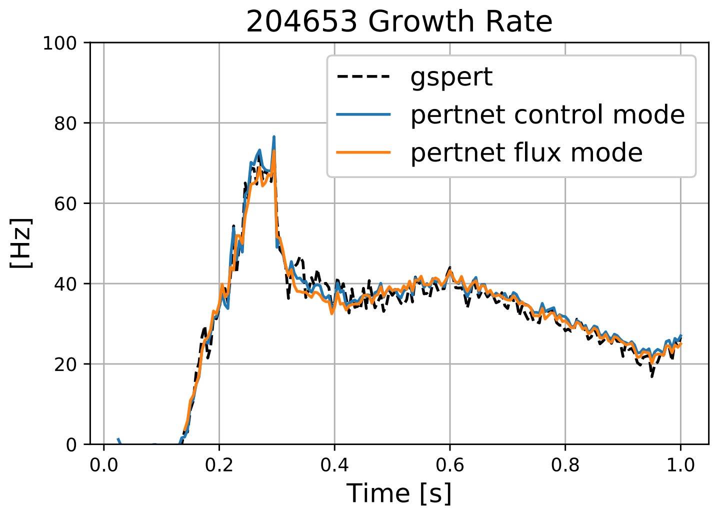
Flux response gives high visual agreement with Gspert code



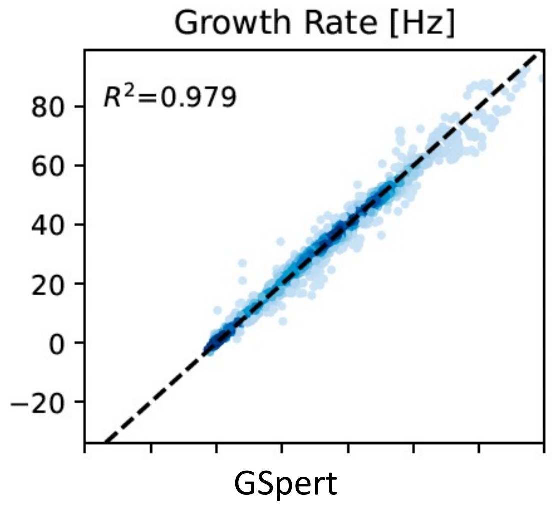


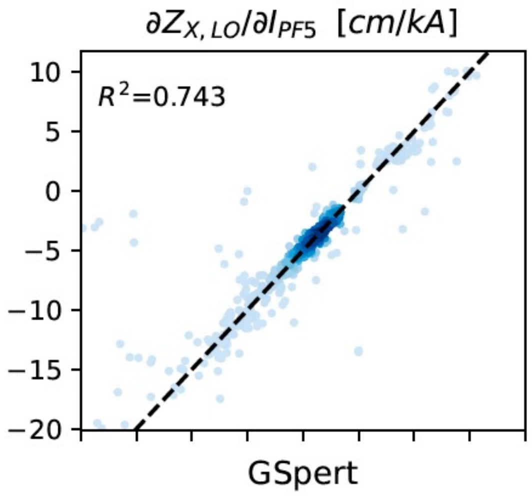
Vertical growth rate predicted within 3Hz deviation from Gspert

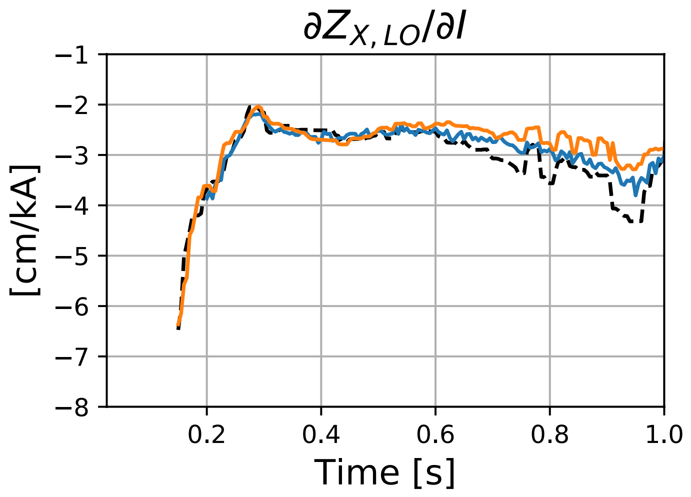
- Neural net captures most trends, some scatter in predictions.
- Again, worst predictions occur during dynamic oscillatory periods of shot. (5% worst predictions all within 200ms of shot start/end)
- RMSE: growth rate=3Hz, dzx/dI = 2.7cm/kA.



Pertnet







Next steps

Conclusion & next steps

- Successfully demonstrated reliability and performance of equilibrium and plasma response neural networks.
- Couple these to profile predictors for *fast* integrated transport+shape simulation capabilities.
- Integrate into optimal planning of feedforward coil current trajectories to achieve a target shape evolution.
- Online shape control updates and vertical stability monitoring – when NSTXU is ready!

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