Control of Advanced Divertors – NSTXU, ITER, D3D

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October 28, 2019

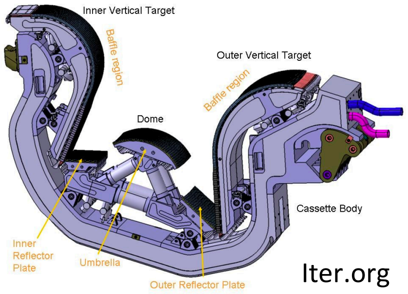
2019 Workshop on MHD Stability Control

Outline

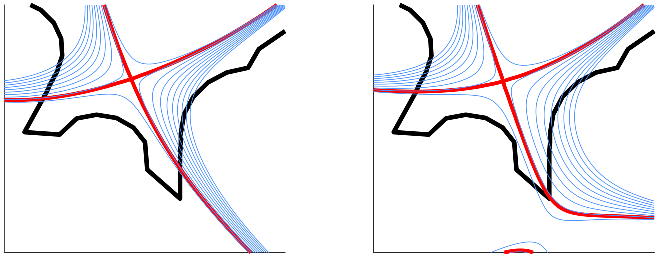
- Motivation for advanced divertors
  - Heat flux spreading
- Dynamics model
- Output model
- **NSTX:** Linear Quadratic Integral (LQI) control of the snowflake divertor
- **ITER:** Model predictive control (MPC) of the X-divertor
- **DIII-D:** Improving snowflake reconstruction with IRTV diagnostic
- **NSTX:** Optimization of the cryopump location for snowflakes

Advanced magnetic field configurations can reduce power flux to the divertors

- Divertor heat load is a design challenge for high performance tokamaks
  - ITER ~ 10 MW/m^2
- Several ideas to reduce heat load
  - Minimize divertor plate angle (but > 1 deg)
  - Strike point sweeping
  - Advanced divertor configurations
    - Snowflake divertor
    - x divertor
    - super x divertor



Iter.org



ITER Standard Divertor

ITER X Divertor

The circuit equation applied to each conductor gives the state-space model dynamics

- Circuit Model

$$v_s = R_s I_s + \dot{\Psi}_{ss,\ coil} + \dot{\Psi}_{ss,\ plasma}$$

$$\dot{\Psi}_{ss,\ coil} = M_{ss} \dot{I}_s$$

Flux change due to induced currents.

$$\dot{\Psi}_{ss,plasma} \approx \left. \frac{\partial \Psi_s}{\partial I_s} \right|_{eq} \delta \dot{I}_s := X_{ss} \delta \dot{I}_s$$

Flux change due to plasma motion. Computed via TokSys and [1]

- State Space Form

$$\begin{bmatrix} \delta v_s \\ 0 \end{bmatrix} = \begin{bmatrix} R_s & 0 \\ 0 & R_p \end{bmatrix} \begin{bmatrix} \delta I_s \\ \delta I_p \end{bmatrix} + \begin{bmatrix} M_{ss} + X_{ss} & M_{sp} + X_{sp} \\ M_{ps} + X_{ps} & M_{pp} + X_{pp} \end{bmatrix} \begin{bmatrix} \delta \dot{I}_s \\ \delta \dot{I}_p \end{bmatrix}$$

$$\Updownarrow$$

$$\delta \dot{I} = A(t) \delta I + B(t) \delta v$$

[1] A.S. Welander et. Al, “Nonrigid, Linear Plasma Response Model Based on Perturbed Equilibria for Axisymmetric Tokamak Control Design,” FST, V47:3, 2005.

The linearized output equation is determined by a derivative expansion of the absolute error

- Controlled outputs

$$Z = \begin{bmatrix} I_p & r_x & z_x & r_{strike} & z_{strike} & \psi_{bry} & \psi_{cp} \times 31 \end{bmatrix}^T$$

- Write the output model in the linearized frame (matches dynamics).

$$e = Z - Z_{target} \qquad \delta e = \frac{\partial (Z - Z_{target})}{\partial I} \delta I \quad \Leftrightarrow \quad y = C(t)\delta I$$

- Reference trajectory defined by setting error to zero

$$0 = e := y + e_0 \quad \Leftrightarrow \quad r = -e0$$

- X-Point response

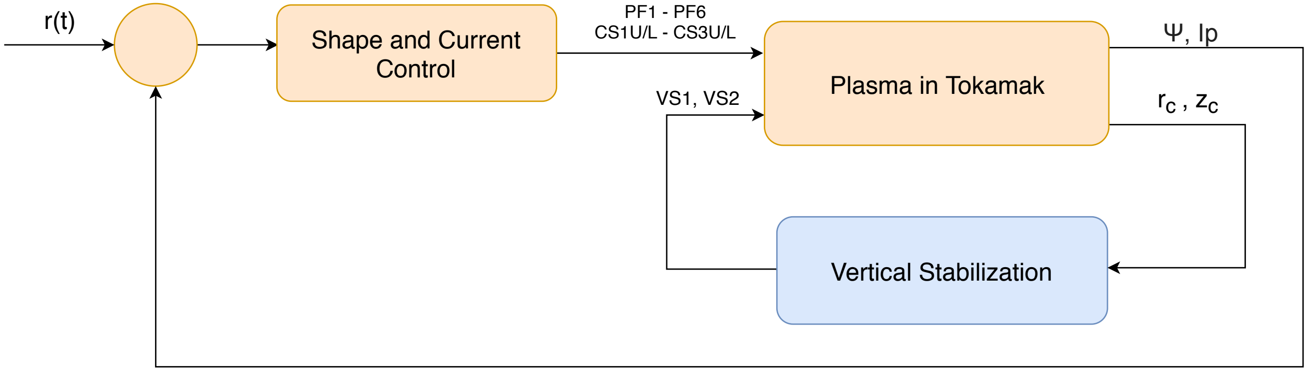
$$\frac{\partial (r_x, z_x)}{\partial I} = \frac{\partial (r_x, z_x)}{\partial (\psi_r, \psi_z)} \frac{\partial (\psi_r, \psi_z)}{\partial I} = \left[-\frac{\partial (\psi_r, \psi_z)}{\partial (r, z)} \right]^{-1} \begin{bmatrix} \partial_r \partial_I \psi \\ \partial_z \partial_I \psi \end{bmatrix}$$

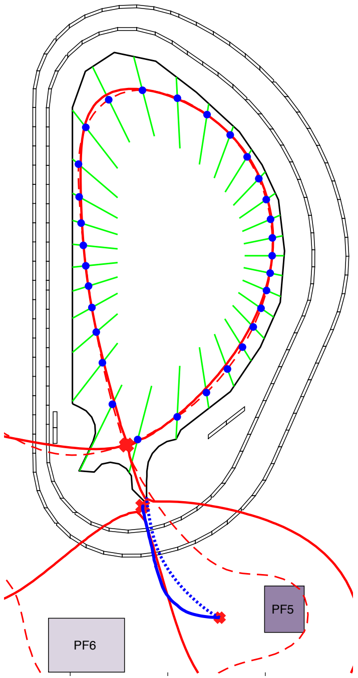
Determined by [1].
Linearization to G-S.

[1] A.S. Welander et. Al, “Nonrigid, Linear Plasma Response Model Based on Perturbed Equilibria for Axisymmetric Tokamak Control Design,” FST, V47:3, 2005.

Since there is a large separation of timescales, current & shape control can be designed separately from vertical stabilization

- Superconducting coil response time (s) **vs.** resistive wall decay time (ms)
- Simulation: negate eigval, exclude use of vs1/vs2 as actuators
- 3 control objectives
  - Minimize flux error between control pts and plasma boundary
  - Reference tracking of x-point positions
  - (ITER) Maintain Ip





NSTXU: Snowflake divertor control on NSTXU can be implemented with a decoupled LQI, proportional controller – P.J. Vail [1]

- Decoupled control scheme:
  - Linear quadratic integral (LQI) for divertor variables
  - Proportional control on isoflux shape
- Reference tracking

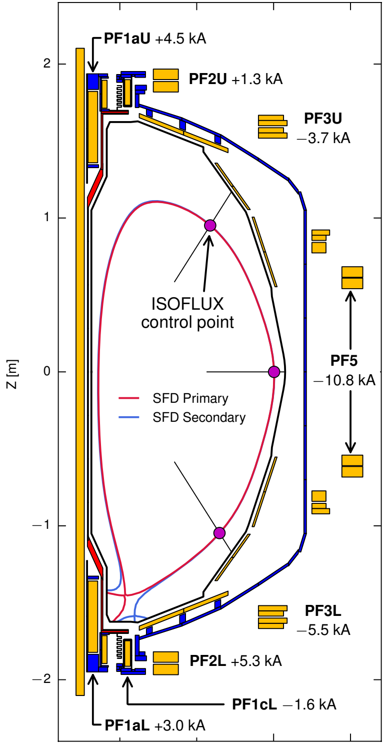
$$\begin{aligned} Ax^* + Bu^* &= 0 \\ Cx^* &= r \end{aligned} \implies \begin{bmatrix} x^* \\ u^* \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix} r := \begin{bmatrix} F_x r \\ F_u r \end{bmatrix}$$

- Final feedback control law

$$u = -K_p(x - F_x r) + F_u r + K_I \int_0^t (y - r) d\tau$$

- Kp and KI from LQR of augmented system

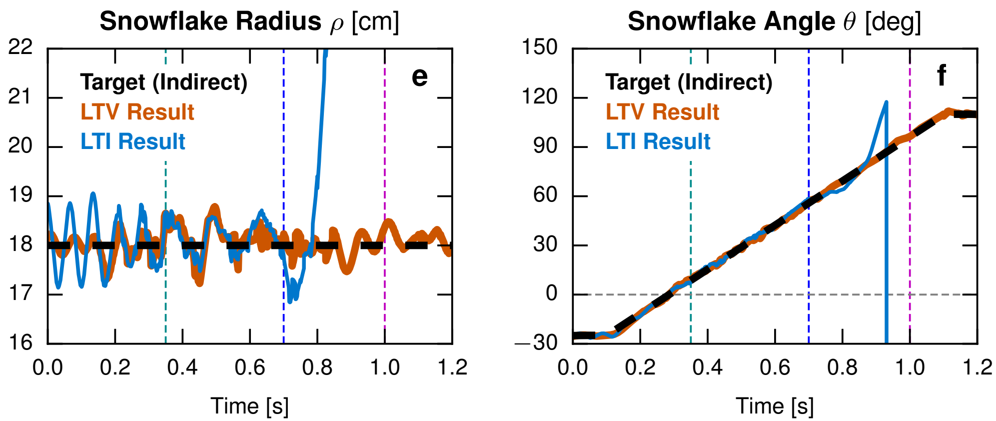
$$\hat{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

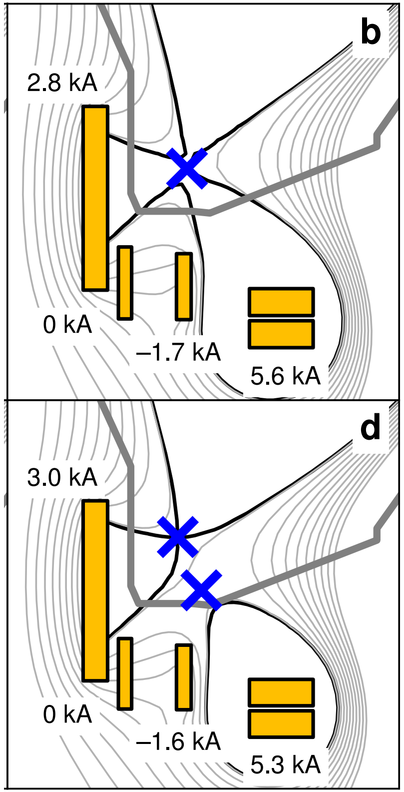


[1] P.J. Vail et Al., "Design and simulation of the snowflake divertor control for NSTX–u," PPCF, V61:3, 2019

NSTXU: Robust snowflake divertor control requires the use of online model updates – P.J. Vail [1]

- Simulation shows high degree of control over snowflake configuration
- Highlights need for online model changes (LTV)

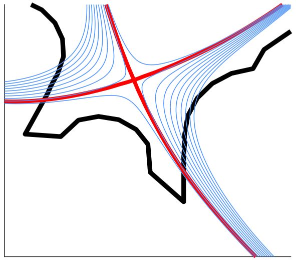


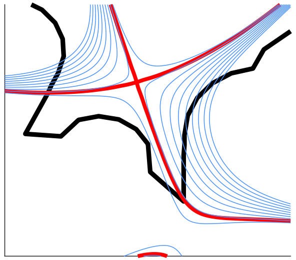


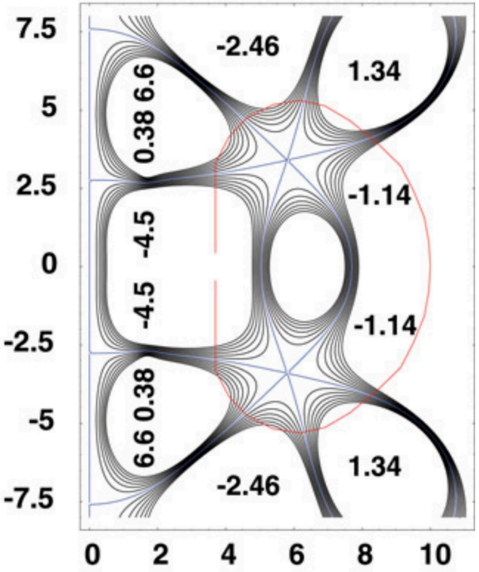
[1] P.J. Vail et Al., “Design and simulation of the snowflake divertor control for NSTX–u,” PPCF, V61:3, 2019

ITER: Out of all advanced divertor configurations, only the X-divertor is physically achievable

- Divertor configurations on ITER
  - Snowflake divertor – exceeds coil currents [1]
  - Super X divertor – geometry changes [2]
  - X divertor possible [2]


ITER Standard Divertor


ITER X Divertor



[1] ITER snowflake equilibria

[1] K. Lackner, H. Zohm, “Calculation of Realistic Snowflake Equilibria for Next-Step Devices”, FST, V63:1, 2013. [2] B. Covele et Al., “An exploration of advanced x-divertor scenarios on ITER,” NF, V54:7, 2014.

ITER: Physical differences on ITER necessitate a more integrated control approach (MPC)

- Poloidal field coils are far away from the plasma, flux effects are more coupled
- No separate set of divertor coils
- Easy to run into coil current constraints
- System is not strictly controllable
  - 12 PF coils but only 11 independent coil circuits
  - 31 shape pts + I_p + Ψ_{bry} + 6 divertor variables = 39 outputs
  - Plus constraint set (35 additional variables)

Activated Constraints				
PF Coils				**Outputs**
Coil #	**I**	**V**		**--**
PF1	< 48 kA	< 1.5 kV		$\dot{P}$ < 200 MW/s
PF2	< 55 kA	< 1.5 kV		P < 250 MW
PF3	< 55 kA	< 1.5 kV		δI_p < 3%
PF4	< 55 kA	< 1.5 kV		r_{strike} on plate
PF5	< 52 kA	< 1.5 kV		z_{strike} on plate
PF6	< 52 kA	< 1.5 kV		cp_{13} gap
CS1U	< 45 kA	< 1.5 kV		cp_{14} gap
CS1L	< 45 kA	< 3.0 kV		cp_{15} gap
CS2U				cp_{16} gap
CS2L	< 45 kA	< 1.5 kV		
CS3U	< 45 kA	< 1.5 kV		
CS3L	< 45 kA	< 1.5 kV		

Red cells affect the control optimization

ITER: MPC optimizes the control inputs over a finite horizon, subject to constraints

- Quadratic cost on the output errors and control actuation

$$J_k = \sum_{i=1}^{N} \left[(y_{k+i} - r_{k+i})^T Q_i (y_{k+i} - r_{k+i}) + u_{k+i-1}^T R_i u_{k+i-1} \right]$$

- Use dynamics model to predict future outputs

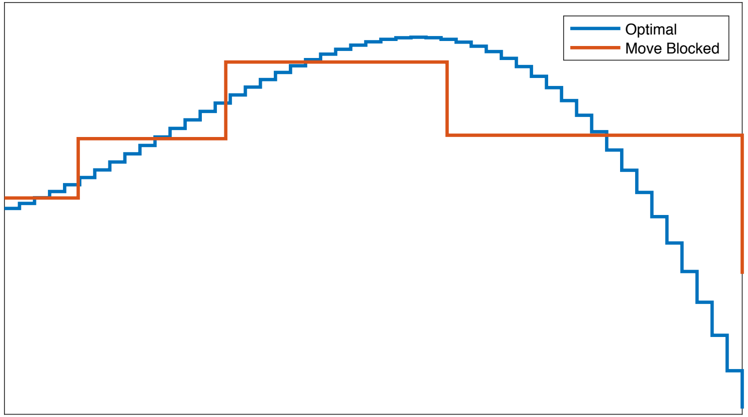
$$x_{k+1} = Ax_k + Bu_k$$
$$y_{k+1} = Cx_{k+1}$$

- After substitution, obtain convex cost function in standard quadratic-program form
  - Solve via ***mpcqpsolver*** in MATLAB

$$J_k = \hat{U}^T H \hat{U} + 2f^T \hat{U} + J_\theta \qquad \hat{U} := \begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+N-1} \end{bmatrix}$$

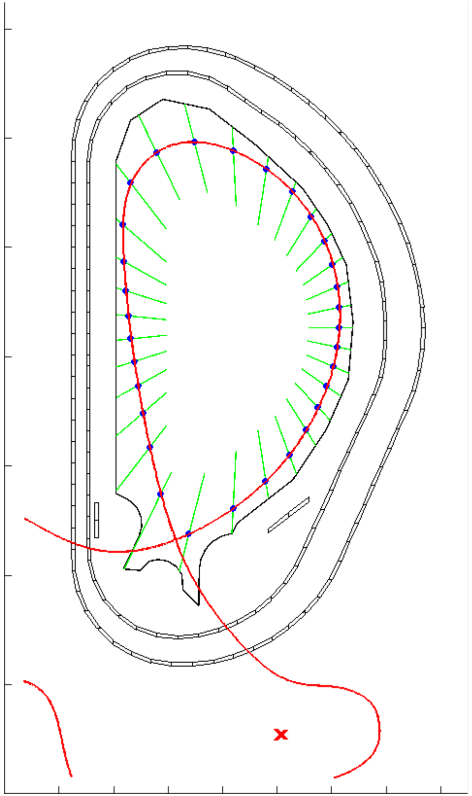
ITER: MPC is computationally intensive, but is expected to be feasible for real-time

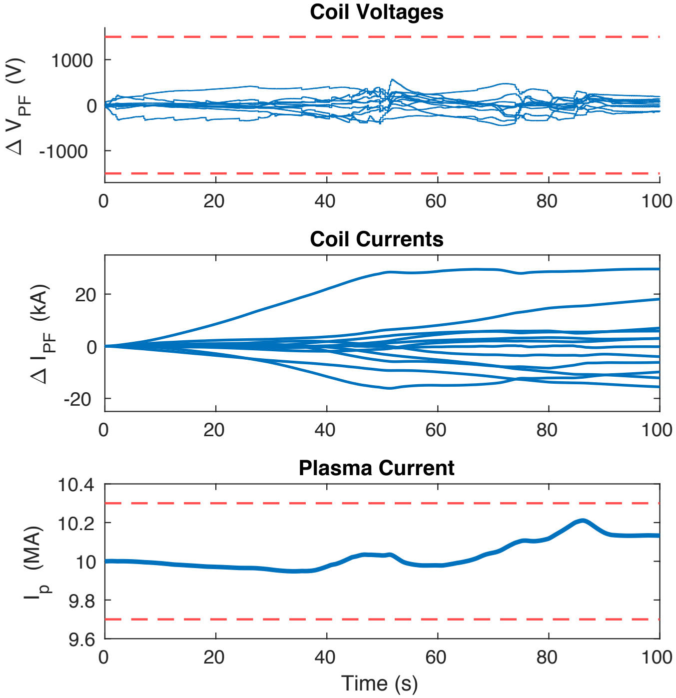
- MPC can be fast (3-7 ms) [1], could be used in real-time
- Several tricks for speeding up simulation
  - Truncated prediction model
    - Neglects vacuum vessel currents
    - (N x 13) **versus** (N x 163)
- Move blocking
  - Reduces the number of optimization variables
  - Geometrically scaling block sizes



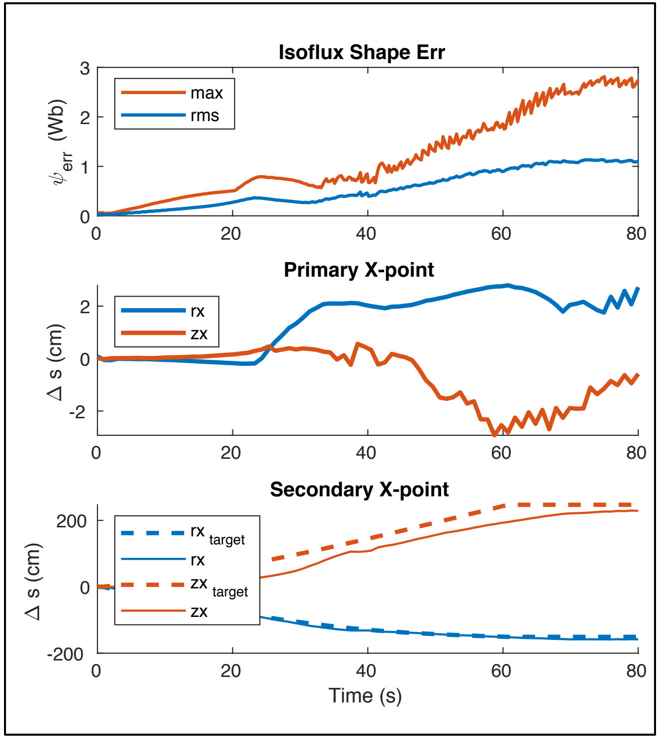
[1] S. Gerksic et Al., “Model predictive control of ITER plasma current and shape using singular-value decomposition,” FED, V129:1, 2018

ITER: X-divertor can be achieved while satisfying constraint set, Ip = 10 MA

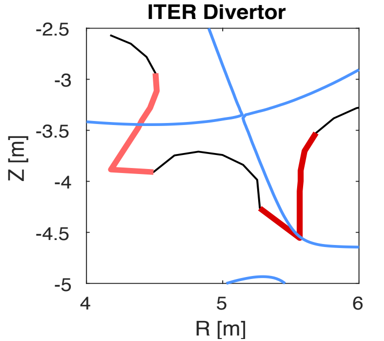


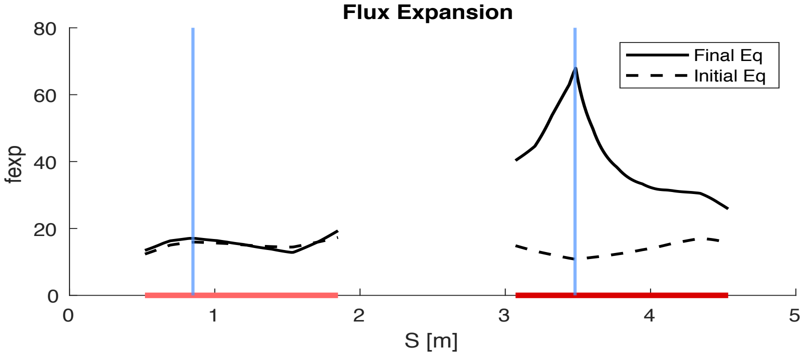


ITER: large changes to the secondary x-point location can be realized with minimal impact on the primary x-point and shape



$$f_{exp} = \frac{(B_p/B)_{mid}}{(B_p/B)_{strike}}$$



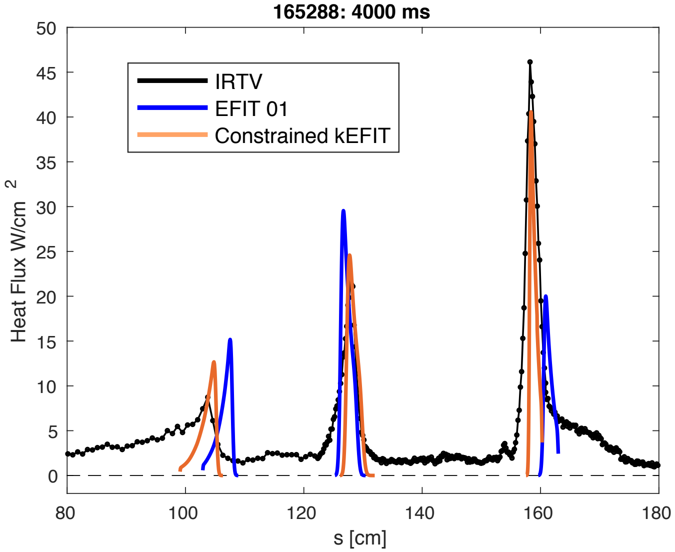


DIII-D: the Infrared TV diagnostic can be used to identify snowflake x-points and better constrain the equilibrium reconstruction – P.J. Vail

- IRTV diagnostic measures heat flux on the divertor plates
- Predicted heat flux of the snowflake equilibrium reconstruction does not match IRTV
  - Opportunity for IRTV to provide additional info to reconstruction algorithm

- **Approach**
  - Analytical model [1]: x point locations --> heat flux
  - ML regression tree: heat flux --> x point locations
  - Use predicted x-points to constrain equilibria

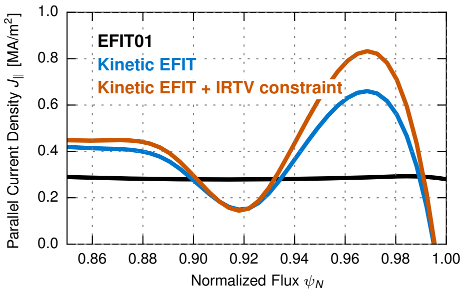
- Constrained equilibria match measured heat flux better

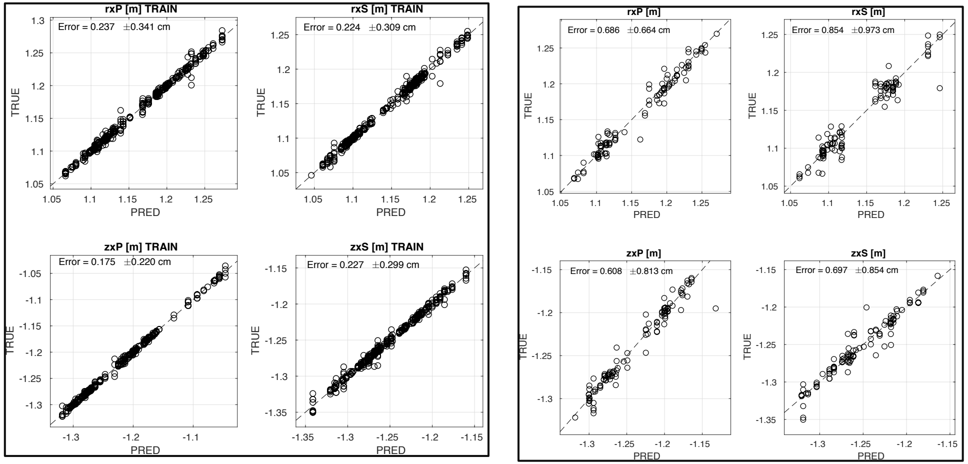


[1] P.J. Vail et Al., "Optimization of the snowflake divertor for power and particle exhaust on NSTX-U," *NME*, V19, 2019.

DIII-D: heat-flux-constrained equilibria reveals 20% difference in edge currents – P.J. Vail

- ML Predictions
  - 17 shots with 25 time slices each
  - ~ 1cm error on the testing data set
- How do the profiles in the heat-flux constrained equilibrium differ?



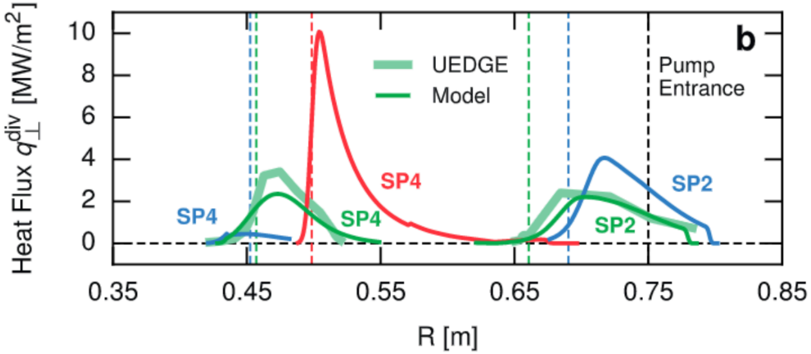


- ~20% difference in edge current. Further studies to perform this analysis across the database and quantify.

NSTXU: for overall divertor performance, the snowflake divertor must work well with the particle exhaust mechanism- P.J. Vail

- Divertor functions: power exhaust ***AND*** particle exhaust
- Does the snowflake divertor work well with conventional particle exhaust (cryopump)?
  - How to optimally place cryopump?

- Analytical model for snowflake power flux
  - Diffusion eqn solved in 2 separate domains, characterizes better than a standard divertor with large flux expansion



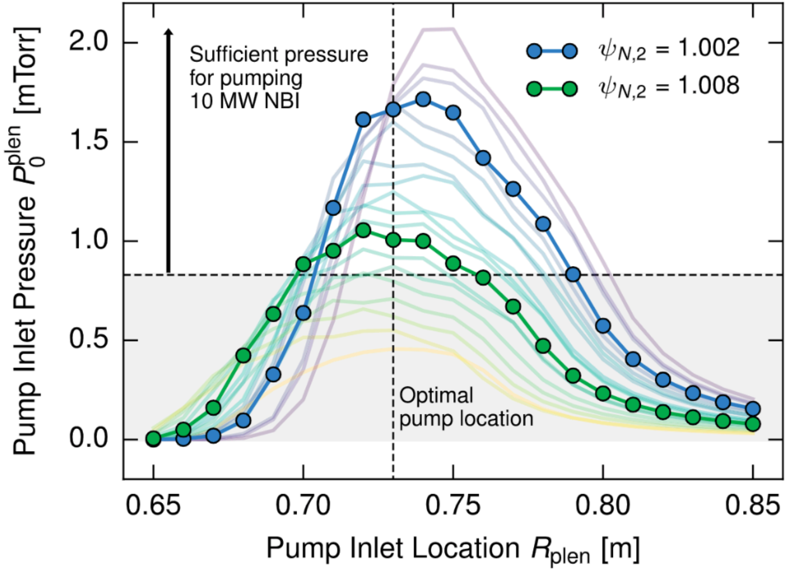
[1] P.J. Vail et Al., "Optimization of the snowflake divertor for power and particle exhaust on NSTX-U," *NME*, V19, 2019.

NSTXU: An optimal cryopump location allows for full power and particle exhaust over a range of snowflakes- P.J. Vail [1]

- Heat flux profile directly related to particle flux profile [2]

$$\Gamma_{\perp}^{div} = q_{\perp}^{div} / \gamma T_e$$

- Assumptions
  - 24 kL/s volumetric pump rate for liquid helium cooled cryopump
  - 10 MW (20 Torr-L/s) of neutral beam heating
  - Gives inlet pressure condition[1,3]: P > 0.83 mTorr



[1] P.J. Vail et Al., "Optimization of the snowflake divertor for power and particle exhaust on NSTX-U," *NME*, V19, 2019.
[2] T. Lunt, et Al., "Numerical study of potential heat flux mitigation effects in the TCV snowflake divertor," PPCF, V58, 2016
[3] M. Ono et Al., "Exploration of spherical torus physics in the NSTX device," NF, V40 (2000)

Summary

- Developing multiple analysis and control tools to improve performance of advanced divertor configurations
- Snowflake divertor control on NSTX can be achieved with high degree of control. Highlights need for online model changes.
- Model predictive control on ITER
  - large changes in the the divertor field geometry can be obtained within the limits of physical constraints
  - It may be possible to create and test the x-divertor on ITER
- IRTV can be used as a diagnostic to improve snowflake equilibrium reconstructions on DIII-D
- Improved UEDGE simulations guide the design of optimal cryopump locations for NSTXU snowflakes
- **Future work**
  - Perform larger analyses of IRTV edge current predictions
  - Implement online model changes for NSTX in order to control ramp-up scenarios (M.D. Boyer, P.J. Vail)